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BOUNDARY LAYER OF A DUAL TEMPERATURE PLASMA ON THE ELECTRODES OF A MAGNETO-HYDRODYNAMIC CHANNEL WITH LATTICE-TYPE ELECTRIC AND MAGNETIC FIELDS AT HIGH VALUES OF THE HALL PARAMETER

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BOUNDARY LAYER OF A DUAL TEMPERATURE PLASMA ON THE ELECTRODES OF A MAGNETO-HYDRODYNAMIC CHANNEL WITH LATTICE-TYPE ELECTRIC AND MAGNETIC FIELDS AT HIGH VALUES OF THE HALL PARAMETER

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ABSTRACT. Existing works on the magneto-hydrodynamics of the boundary layer are concerned with examination of isolated questions and simplified problems, with no complete system of equations for the boundary layer of a compressible magnetized two-temperature plasma on electrodes. Here these equations are developed, limited to an ionized quasineutral plasma at small Reynolds values.

Rational organization of the working process of various types of /40* magneto-hydrodynamic devices requires first of all a knowledge of the laws of interaction of the flow of plasma with electrodes. Therefore the problem of a boundary layer of plasma on the electrodes of an MHD channel represents a matter of considerable practical interest. As a rule, a significant amount of resolution takes place in accelerator channels and the intensity of the magnetic field is fairly high. This leads to the necessity of calculating the fields of Hall currents and the effect of the Hall parameter on the character of plasma transfer. Besides this, plasma accelerators usually exhibit a break in electron temperature.

Existing works on the magneto-hydrodynamic boundary layer are devoted to examination of isolated particular questions or the solution of simplified problems [1-3]. In the literature there is no complete system of equations of the boundary layer of a compressible magnetized two-temperature plasma on electrodes.

In the following, we give the boundary layer equation for the general case indicated. We are limited completely to an ionized quasineutral plasma at small values of Reynolds magnetic number.

We will direct axis x along the conducting wall, axis y along the normal to the wall, axis x perpendicular to axes x and y. Let the external magnetic

*Numbers in the margin indicate pagination in the foreign text.

field be characterized by components $B_x = B_y = 0$, $B_z = B$, and the electric field generated in the plasma by components E_x, E_y.

The system of equations of magnetic gas dynamics for a dense stable flow of plasma will be [4]

$$mn\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+j_yB-\frac{\partial \pi^i_{xx}}{\partial x}-\frac{\partial \pi^i_{xy}}{\partial y} \tag{1}$$

$$mn\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}-j_xB-\frac{\partial \pi^i_{yx}}{\partial x}-\frac{\partial \pi^i_{yy}}{\partial y} \tag{2}$$

$$\frac{3}{2}kn\left(u\frac{\partial T_i}{\partial x}+v\frac{\partial T_i}{\partial y}\right)=-\left(\frac{\partial q_x{}^i}{\partial x}+\frac{\partial q_y{}^i}{\partial y}\right)-p_i\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)-$$
$$-\pi^i_{xx}\frac{\partial u}{\partial x}-\pi^i_{xy}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)-\pi^i_{yy}\frac{\partial v}{\partial y}+3k\frac{m_e}{m}\frac{n}{\tau_e}(T_e-T_i) \tag{3}$$

$$\frac{3}{2}kn\left(u\frac{\partial T_e}{\partial x}+v\frac{\partial T_e}{\partial y}\right)=-\left(\frac{\partial q_x{}^e}{\partial x}+\frac{\partial q_y{}^e}{\partial y}\right)-p_e\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)- \tag{4}$$
$$-\pi^e_{xx}\frac{\partial u}{\partial x}-\pi^e_{xy}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)-\pi^e_{yy}\frac{\partial v}{\partial y}+j_x(E_x+vB)+j_y(E_y-uB)-$$
$$-\frac{3km_e}{m}\frac{n}{\tau_e}(T_e-T_i),\qquad \frac{\partial}{\partial x}(nu)+\frac{\partial}{\partial y}(nv)=0 \tag{5}$$

Here u and v are projections of the mass-center velocity of the plasma along the coordinate axis; n is the numerical density of electrons or ions; p is the overall pressure; p_e, p_i partial pressures; T_e, T_i are electron and ion temperatures; m, m_e are the masses of the ions and electrons respectively.

Projections of the current density are given by a generalized Ohm's Law /41

$$j_x=\sigma\left\{E_x+vB+\frac{1}{en}\frac{\partial p_e}{\partial x}-\frac{A_2}{A_1}\left[E_y-uB+\frac{1}{en}\frac{\partial p_e}{\partial y}\right]+\right.$$
$$\left.+\frac{k}{e}\left(\frac{A_3}{A_1}\frac{\partial T_e}{\partial x}-\frac{A_4}{A_1}\frac{\partial T_e}{\partial y}\right)\right\} \tag{6}$$

$$j_y=\sigma\left\{E_y-uB+\frac{1}{en}\frac{\partial p_e}{\partial y}+\frac{A_2}{A_1}\left(E_x+vB+\frac{1}{en}\frac{\partial p_e}{\partial x}\right)+\right.$$
$$\left.+\frac{k}{e}\left(\frac{A_3}{A_1}\frac{\partial T_e}{\partial y}+\frac{A_4}{A_1}\frac{\partial T_e}{\partial x}\right)\right\} \tag{7}$$

Here σ is the conductivity of the plasma across the magnetic field, A_k is a function of the Hall parameter for H_e electrons. The tensor elements of tensile pressures of ions and electrons have the form

$$
\begin{aligned}
\pi_{xx}^{i} &= -\eta_i\left[\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y} + H_i\frac{5}{3}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right]\\
\pi_{xy}^{i} &= -\eta_i\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - H_i\frac{5}{3}\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)\right]\\
\pi_{yy}^{i} &= -\eta_i\left[\frac{4}{3}\frac{\partial v}{\partial y} - \frac{2}{3}\frac{\partial u}{\partial x} - H_i\frac{5}{3}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right]\\
\pi_{xx}^{e} &= -\eta_e\left[\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y} - H_e\frac{50}{3(2+\sqrt{2})}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right]\\
\pi_{xy}^{e} &= -\eta_e\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + H_e\frac{20}{3(2+\sqrt{2})}\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)\right]\\
\pi_{yy}^{e} &= -\eta_e\left[\frac{4}{3}\frac{\partial v}{\partial y} - \frac{2}{3}\frac{\partial u}{\partial x} + H_e\frac{50}{3(2+\sqrt{2})}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right]
\end{aligned}
\tag{8}
$$

The projections of the thermal flow vectors of electrons and ions will be

$$
\begin{aligned}
q_x^{\,i} &= -\lambda_i\left(\frac{\partial T_i}{\partial x} + \frac{5}{4}H_i\frac{\partial T_i}{\partial y}\right), \quad q_y^{\,i} = -\lambda_i\left(\frac{\partial T_i}{\partial y} - \frac{5}{4}H_i\frac{\partial T_i}{\partial x}\right)\\
q_x^{\,e} &= -\lambda_e\left(\frac{\partial T_e}{\partial x} - \frac{A_6}{A_5}\frac{\partial T_e}{\partial y}\right) - \lambda_e\frac{e}{k}\frac{A_7}{A_5}\left(E_x + vB + \frac{1}{en}\frac{\partial p_e}{\partial x}\right) +\\
&\quad + \lambda_e\frac{e}{k}\frac{A_8}{A_5}\left(E_y - uB + \frac{1}{en}\frac{\partial p_e}{\partial y}\right)\\
q_y^{\,e} &= -\lambda_e\left(\frac{A_6}{A_5}\frac{\partial T_e}{\partial x} + \frac{\partial T_e}{\partial y}\right) - \lambda_e\frac{e}{k}\frac{A_7}{A_5}\left(E_y - uB + \frac{1}{en}\frac{\partial p_e}{\partial y}\right) -\\
&\quad - \lambda_e\frac{e}{k}\frac{A_8}{A_5}\left(E_x + vB + \frac{1}{en}\frac{\partial p_e}{\partial x}\right)
\end{aligned}
\tag{9}
$$

Transfer coefficients will be

$$
\begin{aligned}
\sigma &= \frac{ne^2\tau_e}{m_e}A_1 \qquad \eta_i = \frac{{}^5/_6 kT_i\tau_i n}{1+{}^{25}/_9 H_i^{\,2}}, \quad \lambda_i = \frac{{}^{25}/_8 n\tau_i}{1+{}^{25}/_{16}H_i^{\,2}}\frac{k^2}{m}T_i\\
\eta_e &= \frac{5}{3(2+\sqrt{2})}\frac{kT_e\tau_e n}{1+100/9+(2+\sqrt{2})^2 H_e^{\,2}}, \quad \lambda_e = \frac{5}{2}\frac{nk^2}{m_e}T_e\tau_e A_5
\end{aligned}
\tag{10}
$$

In these expressions

$$
\begin{aligned}
&A_1 = \nabla(1.93 + 1.07H_e^{\,2}), \quad A_2 = \nabla(4.69H_e + 1.07H_e^{\,3})\\
&A_3 = \nabla(1.55 - 1.6H_e^{\,2}), \quad A_4 = \Delta 4.6H_e\\
&A_5 = \nabla(2.58 + 0.391H_e^{\,2}), \quad A_6 = \nabla(6.63H_e + 1.06H_e^{\,3})\\
&A_7 = \nabla(2.55 + 0.42H_e^{\,2}), \quad A_8 = \nabla(6.53H_e + 1.07H_e^{\,3})\\
&\nabla = (1 + 6.73H_e^{\,2} + 1.67H_e^{\,4})^{-1}, \quad H_e = \omega_e\tau_e, \quad H_i = \omega_i\tau_i
\end{aligned}
\tag{11}
$$

Here H_e, H_i are Hall parameters, ω_e, ω_i are Larmorov frequencies, and τ_i, τ_e are average times between collisions of ions with each other and of ions with electrons.

The system of gas-dynamic equations (1) through (5) is completed by the following relationships: /42

electrodynamic

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0, \quad \frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y} = 0 \tag{12}$$

state equations

$$p_i = nkT_i, \quad p_e = nkT_e \tag{13}$$

correlation

$$p = p_i + p_e \tag{14}$$

In order to compare the individual terms with each other we reduce the equations to nondimensional form. We shall use the following scales: linear dimensions x_0, transverse dimensions y_0, linear and transverse velocity components u_0 and v_0, respectively, concentration n_0, temperature T_0, pressure $p_0 = n_0 kT_0$, component of electric field intensity E_0, magnetic field B_0, time between collisions τ_{e0}, τ_{i0}. The scales for the Hall parameters are

$$H_{e0} = \frac{eB_0}{m_e}\tau_{e0}, \quad H_{i0} = \frac{eB_0}{m}\tau_{i0}$$

The scales for functions A_k are determined by the relationships (11) according to scale H_{e0}. The scale for gas-kinetic values σ_0, η_{i0}, η_{e0}, λ_{i0}, λ_{e0} corresponds to the expression in (10) with scales n_0, T_0, τ_{e0}, τ_{i0}, H_{e0}, H_{i0}, A_{10}.

For conversion of the equations to the nondimensional form, we take the following criteria of similarity:

$$R_i = \frac{u_0 m n_0 x_0}{\eta_{i0}}, \quad P_i = \frac{\eta_{i0} c_p}{\lambda_{i0}} \quad \left(c_p = \frac{5}{2}\frac{k}{m}\right),$$

$$R_e = \frac{u_0 m n_0 x_0}{\eta_{e0}}, \quad P_e = \frac{\eta_{e0} c_p}{\lambda_{e0}}, \quad \Phi = \frac{eE_0 x_0}{kT_0}, \quad K = \frac{E_0}{Bu_0}$$

$$M = \frac{u_0}{\sqrt{\varkappa T_0 k/m}}, \quad M^D = \frac{\sigma_0 E_0}{en_0\sqrt{\varkappa T_0 k/m}} \quad \left(\varkappa = \frac{5}{3}\right)$$

Here R_e, R_i are the Reynolds numbers of electrons and ions, respectively; P_e, P_i are the Prandtl numbers of electrons and ions; Φ is the electric field criterion; K = load criterion; M = Mach number; M^D = Mach diffusion number, in which the scale of current is taken as $\sigma_0 E_0$. For the scales of y_0 and v_0 we take the expressions

$$y_0 = \frac{x_0}{\sqrt{R_i}}, \quad v_0 = \frac{u_0}{\sqrt{R_i}}$$

Then (1) taken in the nondimensional form with (7) and (8) will have the form (dots indicate values based on their scales).

$$\begin{aligned}\overline{n}\left(\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right)=&-\frac{\partial\bar{p}}{\partial\bar{x}}\frac{1}{\varkappa M^2}+\bar{B}\bar{n}\bar{\tau}_e\left[\bar{A}_1\left(\overline{E}_y-\bar{u}\bar{B}\frac{1}{K}+\frac{1}{\bar{n}}\frac{\partial\bar{p}_e}{\partial\bar{y}}\frac{\sqrt{\overline{R_i}}}{\Phi}\right)+\right.\\&\left.+\bar{A}_2\frac{A_{20}}{A_{10}}\left(\overline{E}_x+\bar{v}\bar{B}\frac{1}{K\sqrt{\overline{R_i}}}+\frac{1}{\bar{n}}\frac{\partial\bar{p}_e}{\partial\bar{x}}\frac{1}{\Phi}\right)\right]\frac{M^D\Phi}{\varkappa M^3K}+\\&+\bar{B}\bar{n}\bar{\tau}_e\left(\frac{A_{30}}{A_{10}}\bar{A}_3\frac{\partial\overline{T}_e}{\partial\bar{y}}\sqrt{\overline{R_i}}+\frac{A_{40}}{A_{10}}\bar{A}_4\frac{\partial\overline{T}_e}{\partial\bar{x}}\right)\frac{M^D}{\varkappa M^3K}+\\&+\frac{\partial}{\partial\bar{x}}\left\{\overline{\eta}_i\left[\frac{4}{3}\frac{\partial\bar{u}}{\partial\bar{x}}-\frac{2}{3}\frac{\partial\bar{v}}{\partial\bar{y}}+\bar{H}_i\bar{H}_{i0}\frac{5}{3}\left(\frac{\partial\bar{v}}{\partial\bar{x}}\frac{1}{\sqrt{\overline{R_i}}}+\frac{\partial\bar{u}}{\partial\bar{y}}\sqrt{\overline{R_i}}\right)\right]\right\}\frac{1}{R_i}+\\&+\frac{\partial}{\partial\bar{y}}\left\{\overline{\eta}_i\left[\frac{\partial\bar{u}}{\partial\bar{y}}+\frac{\partial\bar{v}}{\partial\bar{x}}\frac{1}{R_i}-\bar{H}_i\bar{H}_{i0}\frac{5}{3}\left(\frac{\partial\bar{u}}{\partial\bar{x}}\frac{1}{\sqrt{\overline{R_i}}}-\frac{\partial\bar{v}}{\partial\bar{y}}\frac{1}{R_i}\right)\right]\right\}\end{aligned}\tag{15}$$

Now examine the case where the Reynolds criterion for ions is much more than the other nondimensional criteria and their combinations M, M^D, Φ, K, H_{i0}, H_{e0}, A_{20}/A_{10} and others, and suppose that we could select the scales in such a way that all the nondimensional values and their gradients were on the order of unity. Then in (15) we can discard members containing the factors $1\sqrt{R_i}$ or $1/R_i$ in comparison to members containing multipliers of unity in comparison with analogous members containing the multiplier $\sqrt{R_i}$. Discarding the corresponding members and reverting to the dimensional variables, we obtain /43

$$mn\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\eta_i\frac{\partial u}{\partial y}\right)+j_yB\tag{16}$$

where density of the transverse flow is

$$j_y=\sigma\left(E_y-uB+\frac{1}{en_e}\frac{\partial p_e}{\partial y}+\frac{A_2}{A_1}E_x+\frac{k}{e}\frac{A_3}{A_1}\frac{\partial T_e}{\partial y}\right)\tag{17}$$

Proceeding analogously with (2) taken with (6) and (8) we obtain

$$\frac{\partial\bar{p}}{\partial\bar{y}}=\bar{B}\bar{n}\bar{\tau}_e\left(\bar{A}_2\frac{A_{20}}{A_{10}}\frac{1}{\bar{n}}\frac{\partial\bar{p}_e}{\partial\bar{y}}+\bar{A}_4\frac{A_{40}}{A_{10}}\frac{\partial\overline{T}_e}{\partial\bar{y}}\right)\frac{M^D}{\varkappa M^3K}\tag{18}$$

Consequently, pressure can be considered constant along the boundary layer if the combination of criteria

$$\frac{M^D}{\varkappa M^3K}=\frac{S}{\Phi}\qquad\left(S=\frac{\sigma_0E_0B_0x_0}{mn_0u_0^2}\right)\tag{19}$$

is sufficiently small. Here S is the parameter of magnetic interaction.

The equation of energy of ions after similar simplifications takes the form

$$mnc_p\left(u\frac{\partial T_i}{\partial x}+v\frac{\partial T_i}{\partial y}\right)=\frac{\partial}{\partial y}\left(\lambda_i\frac{\partial T_i}{\partial y}\right)+u\frac{\partial p_i}{\partial x}+v\frac{\partial p_i}{\partial y}+ \\ +\eta_i\left(\frac{\partial u}{\partial y}\right)^2+\frac{3km_e}{m}\frac{n}{\tau_e}(T_e-T_i) \tag{20}$$

In view of the importance of the equation for electron energy, we will write it out in the full nondimensional form

$$\frac{3}{2}\bar{n}\left(\bar{u}\frac{\partial\bar{T}_e}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{T}_e}{\partial\bar{y}}\right)=\frac{\partial}{\partial\bar{x}}\left\{\bar{\lambda}_e\left(\frac{\partial\bar{T}_e}{\partial\bar{x}}-\sqrt{R_i}\,\frac{A_{60}}{A_{50}}\frac{\bar{A}_6}{\bar{A}_5}\frac{\partial\bar{T}_e}{\partial\bar{y}}\right)\frac{\varkappa}{(\varkappa-1)R_e}\frac{1}{P_e}+\right. \\ +\frac{5}{2}\bar{\sigma}\bar{T}_e\frac{\bar{A}_7}{\bar{A}_1}\left(\bar{E}_x+\bar{v}\bar{B}\,\frac{1}{K\sqrt{R_i}}+\frac{1}{\bar{n}}\frac{\partial p_e}{\partial\bar{x}}\frac{1}{\Phi}\right)\frac{M^D}{M}\frac{A_{70}}{A_{10}}- \\ \left.-\frac{5}{2}\bar{\sigma}\bar{T}_e\frac{\bar{A}_8}{\bar{A}_1}\left(\bar{E}_y-\bar{u}\bar{B}\frac{1}{K}+\frac{1}{\bar{n}}\frac{\partial p_e}{\partial\bar{y}}\frac{\sqrt{R_i}}{\Phi}\right)\frac{M^D}{M}\frac{A_{80}}{A_{10}}\right\}+ \\ +\frac{\partial}{\partial\bar{y}}\left\{\bar{\lambda}_e\left(\frac{\bar{A}_6}{\bar{A}_5}\frac{A_{60}}{A_{50}}\frac{\partial\bar{T}_e}{\partial\bar{x}}\frac{1}{\sqrt{R_i}}+\frac{\partial\bar{T}_e}{\partial\bar{y}}\right)\frac{\varkappa}{\varkappa-1}\frac{R_i}{P_eR_e}+\frac{5}{2}\bar{\sigma}\bar{T}_e\frac{\bar{A}_7}{\bar{A}_1}\left(\bar{E}_y-\bar{u}\bar{B}\frac{1}{K}+\right.\right. \\ \left.+\frac{1}{\bar{n}}\frac{\partial\bar{p}_e}{\partial\bar{y}}\frac{\sqrt{R_i}}{\Phi}\right)\frac{M^D}{M}\frac{A_{70}}{A_{10}}\sqrt{R_i}+\frac{5}{2}\bar{\sigma}\bar{T}_e\frac{\bar{A}_8}{\bar{A}_1}\left(\bar{E}_x+\bar{v}\bar{B}\frac{1}{K\sqrt{R_i}}+\right. \\ \left.\left.+\frac{1}{\bar{n}}\frac{\partial\bar{p}_e}{\partial\bar{x}}\frac{1}{\Phi}\right)\frac{M^D}{M}\frac{A_{80}}{A_{10}}\sqrt{R_i}\right\}-\bar{p}_e\left(\frac{\partial\bar{u}}{\partial\bar{x}}+\frac{\partial\bar{v}}{\partial\bar{y}}\right)+\frac{\partial\bar{u}}{\partial\bar{x}}\left\{\bar{\eta}_e\left[\frac{4}{3}\frac{\partial\bar{u}}{\partial\bar{x}}-\frac{2}{3}\frac{\partial\bar{v}}{\partial\bar{y}}-\right.\right. \\ \left.-\bar{H}_e\frac{10H_{e0}}{3(2+\sqrt{2})}\left(\frac{\partial\bar{u}}{\partial\bar{y}}\sqrt{R_i}+\frac{\partial\bar{v}}{\partial\bar{x}}\frac{1}{\sqrt{R_i}}\right)\right]\left.\right\}\frac{\varkappa M^2}{R_e}+\left(\frac{\partial\bar{u}}{\partial\bar{y}}\sqrt{R_i}+\frac{\partial\bar{v}}{\partial\bar{x}}\frac{1}{\sqrt{R_i}}\right)\times \\ \times\left\{\bar{\eta}_e\left[\frac{\partial\bar{u}}{\partial\bar{y}}\sqrt{R_i}+\frac{\partial\bar{v}}{\partial\bar{x}}\frac{1}{\sqrt{R_i}}+\bar{H}_e\frac{10H_{e0}}{3(2+\sqrt{2})}\left(\frac{\partial\bar{u}}{\partial\bar{x}}-\frac{\partial\bar{v}}{\partial\bar{y}}\right)\right]\right\}\frac{\varkappa M^2}{R_e}+ \\ +\frac{\partial\bar{v}}{\partial\bar{y}}\left\{\eta_e\left[\frac{4}{3}\frac{\partial\bar{v}}{\partial\bar{y}}-\frac{2}{3}\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{H}_e\frac{10H_{e0}}{3(2+\sqrt{2})}\left(\frac{\partial\bar{u}}{\partial\bar{y}}\sqrt{R_i}+\frac{\partial\bar{v}}{\partial\bar{x}}\frac{1}{\sqrt{R_i}}\right)\right]\right\}\frac{\varkappa M^2}{R_e}+ \\ +\left(\bar{E}_x+\bar{v}\bar{B}\frac{1}{K\sqrt{R_i}}\right)\bar{\sigma}\left\{\left(\bar{E}_x+\bar{v}\bar{B}\frac{1}{K\sqrt{R_i}}+\frac{1}{\bar{n}}\frac{\partial\bar{p}_e}{\partial\bar{x}}\frac{1}{\Phi}\right)\frac{\Phi M^D}{M}-\right. \\ -\frac{\bar{A}_2}{\bar{A}_1}\left(\bar{E}_y-\bar{u}\bar{B}\frac{1}{K}+\frac{1}{\bar{n}}\frac{\partial\bar{p}_e}{\partial\bar{y}}\frac{\sqrt{R_i}}{\Phi}\right)\frac{\Phi M^D}{M}\frac{A_{20}}{A_{10}}+\left(\frac{\bar{A}_3}{\bar{A}_1}\frac{\partial\bar{T}_e}{\partial\bar{x}}\frac{A_{30}}{A_{10}}-\right. \\ \left.\left.-\frac{\bar{A}_4}{\bar{A}_1}\frac{\partial\bar{T}_e}{\partial\bar{y}}\frac{A_{40}}{A_{10}}\sqrt{R_i}\right)\frac{M^D}{M}\right\}+(\bar{E}_y-\bar{u}\bar{B})\bar{\sigma}\left\{(\bar{E}_y-\bar{u}\bar{B}+\right. \\ +\frac{1}{\bar{n}}\frac{\partial\bar{p}_e}{\partial\bar{y}}\frac{\sqrt{R_i}}{\Phi}\Big)\frac{\Phi M^D}{M}+\frac{\bar{A}_2}{\bar{A}_1}\left(\bar{E}_x+\bar{v}\bar{B}\frac{1}{K\sqrt{R_i}}+\frac{1}{\bar{n}}\frac{\partial\bar{p}_e}{\partial\bar{x}}\frac{1}{\Phi}\right)\frac{\Phi M_D}{M}\frac{A_{20}}{A_{10}}+ \\ \left.+\left(\frac{\bar{A}_3}{\bar{A}_1}\frac{\partial\bar{T}_e}{\partial\bar{y}}\frac{A_{30}}{A_{10}}\sqrt{R_i}+\frac{\bar{A}_4}{\bar{A}_1}\frac{\partial\bar{T}_e}{\partial\bar{x}}\frac{\bar{A}_{40}}{A_{10}}\right)\frac{M^D}{M}\right\}-3\frac{\bar{n}}{\bar{\tau}_e}(\bar{T}_e-\bar{T}_i)\frac{\Phi A_{10}}{\varkappa MM^D} \tag{21}$$

We will consider the criterion for R_e sufficiently high, so that /44

$$R_e P_e \gtrsim R_i, \quad R_e > H_{e0}\sqrt{R_i}, \quad H_{e0} < \sqrt{R_i} \tag{22}$$

For this the Hall numbers for electrons can also be high, but not unlimited.

Disregarding members containing multipliers of one or $\sqrt{R_i}$ by comparison with analogous members containing corresponding multipliers $\sqrt{R_i}$ and R_i, and calculating inequality (22) returning to dimensional variables and introducing thermal capacity c_p we obtain

$$mnc_p\left(u\frac{\partial T_e}{\partial x}+v\frac{\partial T_e}{\partial y}\right)=\frac{\partial}{\partial y}\left(\lambda_e\frac{\partial T_e}{\partial y}+\frac{5}{2}\frac{j_y^q}{e}kT_e\right)+u\frac{\partial p_e}{\partial x}+ \\ +v\frac{\partial p_e}{\partial y}+j_xE_x+j_yE_y+\eta_e\left(\frac{\partial u}{\partial y}\right)^2-3k\frac{m_e}{m}\frac{n}{\tau_e}(T_e-T_i) \tag{23}$$

Here

$$j_x=\sigma\left\{E_x-\frac{A_2}{A_1}\left(E_y-uB+\frac{1}{en}\frac{\partial p_e}{\partial y}\right)-\frac{k}{e}\frac{A_1}{A_1}\frac{\partial T_e}{\partial y}\right\} \tag{24}$$

is the density of axial (Hall) current, while

$$j_y^q=\sigma\left\{\frac{A_7}{A_1}\left(E_y-uB+\frac{1}{en}\frac{\partial p_e}{\partial y}\right)+\frac{A_8}{A_1}E_x\right\} \tag{25}$$

is the current-carrying density (enthalpy) in the transverse direction.

The nondimensional form of the electrodynamic equations

$$\frac{\partial \bar{E}_y}{\partial \bar{x}}-\frac{\partial \bar{E}_x}{\partial \bar{y}}\sqrt{R_i}=0, \quad \frac{\partial \bar{j}_x}{\partial \bar{x}}+\frac{\partial \bar{j}_y}{\partial \bar{y}}\sqrt{R_i}=0$$

allows us to conclude that E_x and j_y are constant across the boundary layer.

The problem of boundary layer plasma on electrodes will be completely determined if we assign the following boundary conditions:

$$u=v=0, \quad T_i=T_{iw}(x), \quad T_e=T_{ew}(x) \quad \text{for } \bar{y}=\infty \tag{26}$$

$$u=U_s(x), \quad T_i=T_{is}(x), \quad T_e=T_{es}(x), \quad j_y=j_{ys}(x), \quad E_x=E_{xs}(x) \tag{27}$$

One possible method of integrating the system of equations of the boundary layer of a two-temperature plasma is the method of finding local-automodel currents, in which the dependence of all values on the linear

coordinate is calculated by the change of parameters in the external current. /45

Introducing independent variables

$$\xi = \int_0^{\bar{x}} \overline{U}_s d\bar{x}, \quad \zeta = \frac{\overline{U}_s \sqrt{R_i}}{\sqrt{2\xi}} \int_0^{\bar{y}} \bar{n} d\bar{y}$$

$$\left(\overline{U}_s = \frac{U_s}{U_{s0}}, \ \bar{n} = \frac{n}{n_s}, \ \bar{x} = \frac{x}{L}, \ \bar{y} = \frac{y}{L}, \ R_i = \frac{U_{s0} L m n_{s0}}{\eta_{is0}}\right)$$

(the subscript 0 refers to the initial section x = 0) and functions

$$f' = \frac{u}{U_s}, \ \theta_e = \frac{T_e}{T_{es}}, \ \theta_i = \frac{T_i}{T_{is}}$$

We will limit ourselves to examination of the isothermal accelerating external flow with constant parameters

$$T_{i_s} = \text{const}, \ T_{es} = \text{const}, \ p = \text{const}, \ E_{x_s} = \text{const}, \ j_{y_s} = \text{const}, j_{x_s} = 0$$

Then we bring equations (16), (20), and (23) to the form

$$ff'' + \frac{2\xi S \overline{T}_{es}}{\overline{U}_s^3}\theta_e + \frac{2\xi S \overline{T}_{is}}{\overline{U}_s^3}\theta_i - \frac{2\xi}{\overline{U}_s}\frac{d\overline{U}_s}{d\xi} f'^2 + (\overline{\eta}_{i1} f'')' = 0 \tag{28}$$

$$f\theta_i' + (\varkappa - 1) M^2 \frac{\overline{U}_s^2}{\overline{T}_{is}} \bar{\eta}_{i1} f''^2 + (\bar{\lambda}_{i1}\theta_i')' + \frac{\varkappa - 1}{\varkappa} \overline{T}_{es} f \frac{\theta_i\theta_e' - \theta_e\theta_i'}{\theta_e\overline{T}_{es} + \theta_i\overline{T}_{is}} + $$
$$+ 6\frac{\varkappa - 1}{\varkappa^3}\frac{\Phi^2\xi}{M^4 SK}\frac{A_{1s0}^2 + A_{2s0}^2}{\overline{U}_s^2 A_{2s0}\overline{T}_{is}}\frac{\theta_e\overline{T}_{es} - \theta_i\overline{T}_{is}}{\theta_e^{3/2}(\theta_e\overline{T}_{es} + \theta_i\overline{T}_{is})} = 0 \tag{29}$$

$$f\theta_e' + (\varkappa - 1) M^2 \frac{\overline{U}_s^2}{\overline{T}_{es}} \bar{\eta}_{e1} f''^2 + (\bar{\lambda}_{e1}\theta_e')' + \frac{\varkappa - 1}{\varkappa}\overline{T}_{is} f \frac{\theta_e\theta_i' - \theta_i\theta_e'}{\theta e\overline{T}_{es} + \theta_i\overline{T}_{is}} +$$
$$+ 2(\varkappa - 1)\frac{M^2 SK}{\overline{T}_{es}}\frac{\xi}{\overline{U}_s^2}(\theta_e\overline{T}_{es} + \theta_i\overline{T}_{is})\left[\bar{\sigma}\left(1 + \frac{A_2^2}{A_1^2}\right) - 2\frac{A_2}{A_1} + \frac{1}{\bar{\sigma}}\right] +$$
$$+ (\varkappa - 1)\sqrt{2\xi R_i}\frac{M^2 SK}{\Phi U_s}\left\{\theta_e'\left[-\frac{\theta_i\overline{T}_{is}}{\theta_e\overline{T}_{es} + \theta_i\overline{T}_{is}} + \bar{\sigma}\left(\frac{A_2 A_3}{A_1^2} - \frac{A_4}{A_1}\right) + \right.\right.$$
$$\left.\left. + \frac{5}{2}\frac{j_y^q}{j_y} - \frac{A_3}{A_1}\right] + \theta_i'\frac{\theta_e\overline{T}_{is}}{\theta_e\overline{T}_{es} + \theta_i\overline{T}_{is}}\right\} - 6\frac{(\varkappa - 1)\Phi^2\xi}{\varkappa^3 M^4 SK}\frac{A_{1s0}^2 + A_{2s0}^2}{\overline{U}_s^2 A_{2s0}\overline{T}_{es}} \times$$
$$\times \frac{\theta_e\overline{T}_{es} - \theta_i\overline{T}_{is}}{\theta_e^{3/2}(\theta_e\overline{T}_{es} + \theta_i\overline{T}_{is})} = 0 \tag{30}$$

The characteristics of plasma transfer

$$\bar{\eta}_{i1} = \frac{\eta_i/\eta_{is}}{\theta_e\overline{T}_{es} + \theta_i\overline{T}_{is}}, \quad \bar{\lambda}_{i1} = \frac{\bar{\lambda}_i/\lambda_{is}}{(\theta_e\overline{T}_{es} + \theta_i\overline{T}_{is})P_i}, \quad \bar{\eta}_{e1} = \frac{\eta_e/\eta_{es}}{\theta_e\overline{T}_{es} + \theta_i\overline{T}_{is}}\frac{R_i}{R_e},$$
$$\bar{\lambda}_{e1} = \frac{\lambda_e/\lambda_{es}}{(\theta_e\overline{T}_{es} + \theta_i\overline{T}_{is})}\frac{R_i}{P_e R_e}, \quad \bar{\sigma} = \frac{\sigma}{\sigma_{00}} = \frac{\sigma E_x}{j_y} = \frac{\sigma}{\sigma_s}\frac{A_{2s}/A_{1s}}{1 + A_{2s}^2/A_{1s}^2}$$

according to the relationship (10) and the calculation of the determination [4]

$$\tau_i = (4\pi\varepsilon)^2 \frac{3\sqrt{m}\,(kT_i)^{3/2}}{4\sqrt{\pi}\lambda_i e^4 n}, \quad \tau_e = (4\pi\varepsilon)^2 \frac{3\sqrt{m_e}\,(kT_e)^{3/2}}{4\sqrt{2\pi}\lambda_e e^4 n}$$

are expressed by θ_e, θ_i, H_e, H_i, where

$$H_i = \theta_i^{3/2}(\theta_e \bar{T}_{es} + \theta_i \bar{T}_{is})\, H_{is}, \quad H_e = \theta_e^{3/2}(\theta_e \bar{T}_{es} + \theta_i \bar{T}_{is})\, H_{es}$$

In the proposition of the constant j_y^q we can obtain

$$\frac{i_y^q}{i_y} = \frac{A_{1s}A_{2s} + A_{2s}A_{8s}}{A_{1s}^2 + A_{2s}^2}$$

The criteria of comparison in the problem will be the following values: /46

$$S = \frac{i_y BL}{mn_s U_{s0}^2}, \quad M = \frac{U_{s0}}{\sqrt{\varkappa kT_s/m}}, \quad \Phi = \frac{eE_x L}{kT_s}$$

$$K = \frac{E_x}{U_{s0}B}, \quad \bar{T}_{es} = \frac{T_{es}}{T_s}, \quad \bar{T}_{is} = \frac{T_{is}}{T_s}$$

$$R_i = \frac{U_{s0}mn_s L}{\eta_{is}}, \quad R_e = \frac{U_{s0}mn_s L}{\eta_{es}}, \quad P_e = \frac{\eta_{es}c_p}{\lambda_{es}}$$

$$P_i = \frac{\eta_{is}c_p}{\lambda_{is}}, \quad H_{is} = \omega_i \tau_{is}, \quad H_{es} = \omega_e \tau_{es}$$

Note that the parameter of magnetic interaction is related to the diffusion (flow) Mach number by the correlation

$$S = \frac{M^j \Phi}{\varkappa M^3 K}, \quad M_j = \frac{i_y}{en_s\sqrt{\varkappa kT_s/m}} = \frac{\sigma_{00}E_x}{en_s\sqrt{\varkappa kT_s/m}} = \frac{\sigma_{00}}{\sigma_s} M^D$$

The boundary conditions of the problem have the form

$$\begin{aligned} &f(0) = f'(0) = 0, \quad f'(\infty) = 1 \\ &\theta_e(0) = \theta_{ew} = \text{const}, \quad \theta_e(\infty) = 1 \\ &\theta_i(0) = \theta_{iw} = \text{const}, \quad \theta_i(\infty) = 1 \end{aligned} \tag{31}$$

As an example, Figures 1-3 contain the distribution of u/U_s, Θ_i and Θ_e, in the boundary layer of completely ionized two-temperature plasma of argon for $\xi = 0$, obtained from numerical calculation on an M-20 computer using the following values of defining parameters:

$M = 2.45$, $K = 1.036$, $\bar{T}_{es} = 0.833$, $\bar{T}_{is} = 0.167$
$R_i = 1.565 \cdot 10^4$, $R_e = 3.61 \cdot 10^{10}$, $P_i = 0.619$, $P_e = 1.925 \cdot 10^{-5}$
$H_{is} = 0.265$, $H_{es} = 438$, $\Theta_{ew} = 0.15$, $\Theta_{iw} = 0.15$

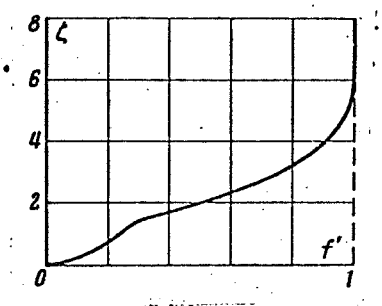

Figure 1.

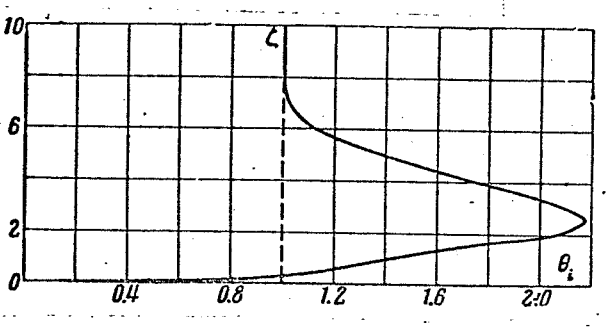

Figure 2.

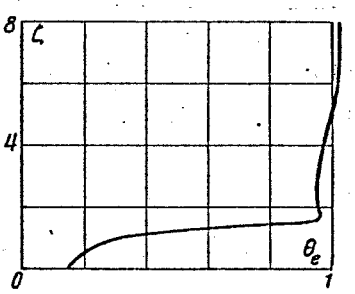

Figure 3.

Certain peculiarities observed in the curves obtained are apparently related to behavior of transfer characteristics in the strong magnetic field.

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